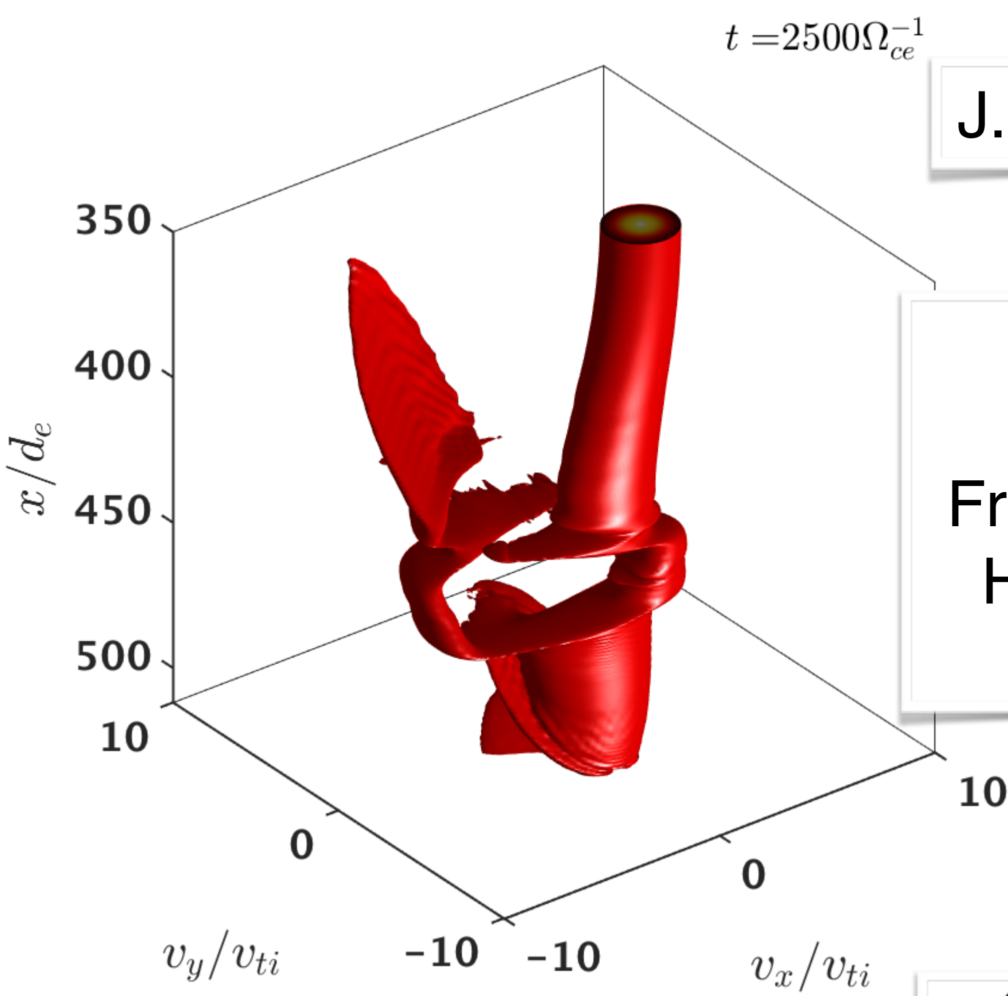
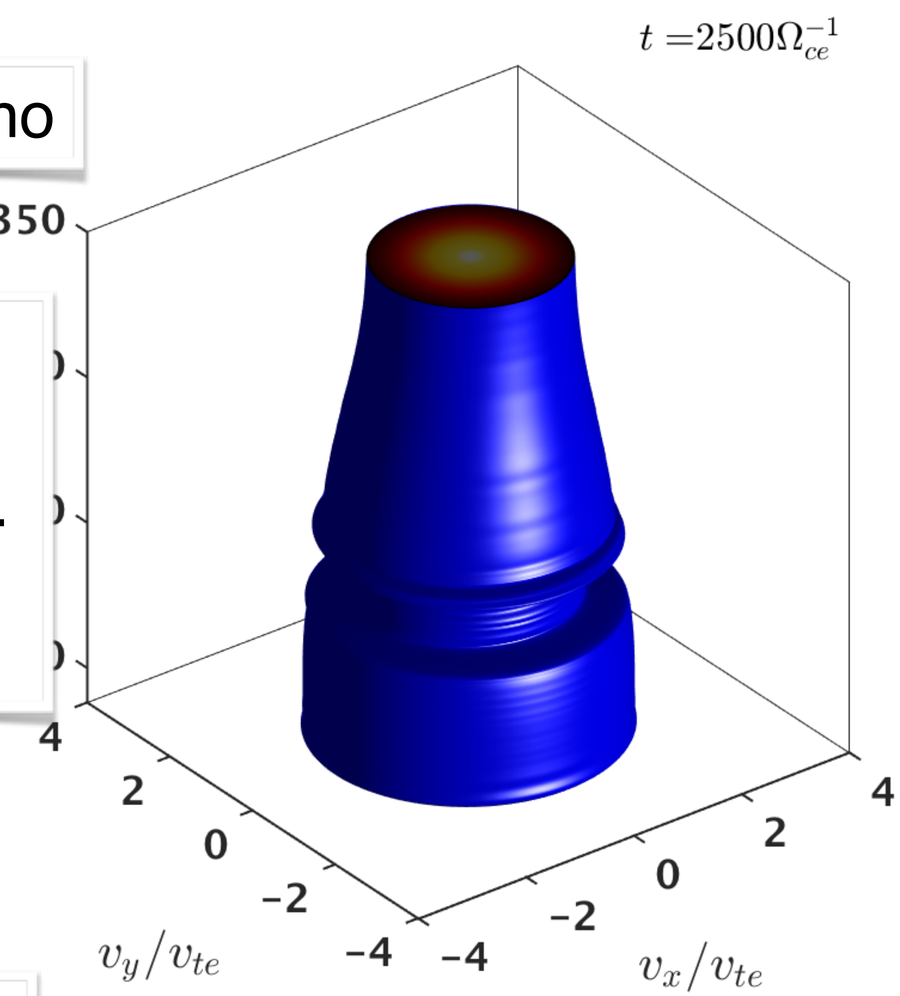
# Energy Dissipation and Phase-Space Dynamics in Eulerian Vlasov-Maxwell Shocks and Reconnection





J. M. TenBarge and J. Juno


<u>Collaborators</u>

A. Bhattacharjee, M. Francisquez, A. Hakim, G. Hammett, G. Howes, K. Klein

February 2019

PRINCETON UNIVERSITY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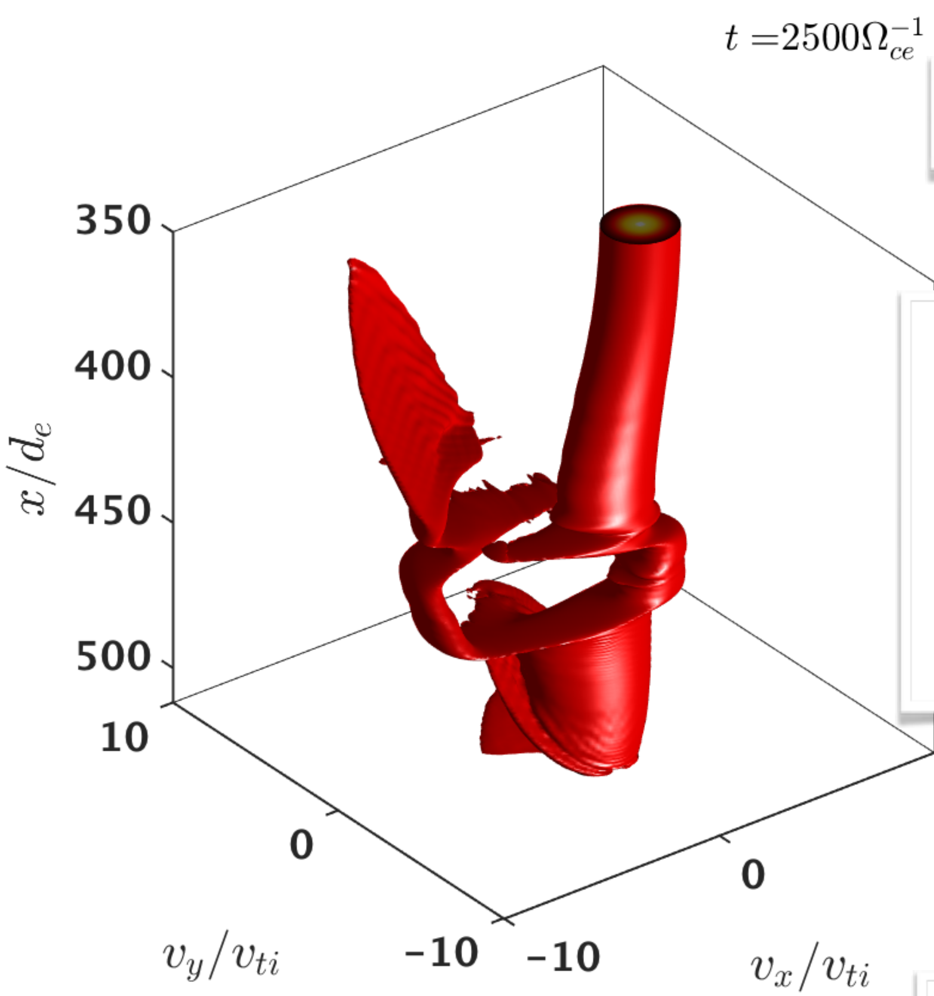


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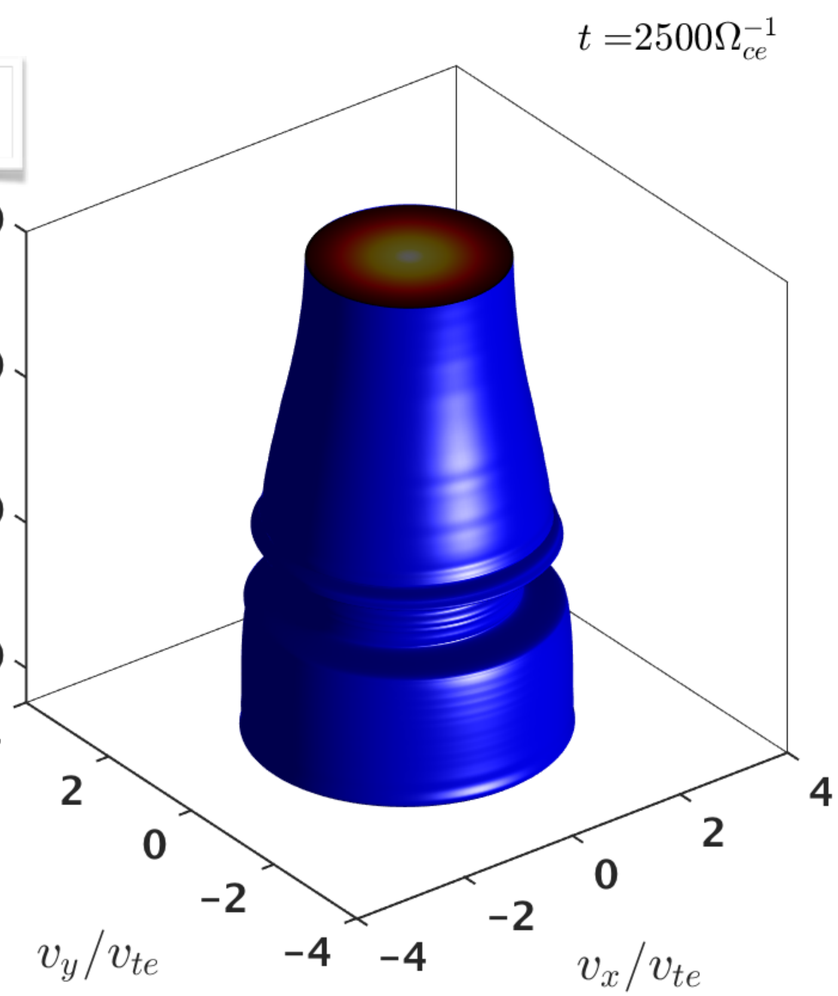


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# Outline

- Introductory material
  - Describe Gkeyll and numerical method
  - Collision operator and magnetic pumping
  - Field-particle correlations

- Non-relativistic, collisionless transverse shock
  - Particle heating
  - Entropy production

# Gkeyll Simulation Framework

# The Gkeyll (and Hyde) Framework*

*"It is one thing to mortify curiosity, another to conquer it."*

- The Gkeyll framework is flexible suite of solvers for plasma physics being developed at the Princeton Plasma Physics Lab, UMD, Virginia Tech, and MIT

- Solvers include a finite volume method for equations written in conservative form and a discontinuous Galerkin finite element method for systems of equations which can be written in terms of a Poisson bracket

- Multiple Vlasov-Maxwell publications already:

  - P. Cagas, A. Hakim, J. Juno, B. Srinivasan, Continuum kinetic and multi-fluid simulation of a classical sheath, *Phys. Plasmas* (2017).
  - P. Cagas, A. Hakim, B. Srinivasan, Nonlinear saturation of the Weibel instability, *Phys. Plasmas* (2017).
  - J. Juno, A. H. Hakim, J. M. TenBarge, B. Dorland, E. L. Shi, Discontinuous Galerkin algorithms for fully kinetic plasmas. *JCP* (2018).
  - I. Pusztai, J. M. TenBarge, A. N. Csapo, J. Juno, A. Hakim, L. Yi, T. Fulop, Low Mach-number collisionless electrostatic shocks and associated ion acceleration, PPCF (2018)
  - V. Skoutnev, A. Hakim, J. Juno, J. M. TenBarge, Temperature Dependent Saturation of Weibel Type Instabilities in Counter-Streaming Plasmas, ApJL in press
  - A. Sundstrom, J. Juno, J. M. TenBarge, and I. Pusztai. Effect of a weak ion collisionality on the dynamics of kinetic electrostatic shocks, JPP (2019)
  - J. Juno and A. Hakim, Generating a Quadrature and Matrix-free Discontinuous Galerkin algorithm for (plasma) kinetic equations, In preparation.
  - A. Hakim, M. Francisquez, J. Juno, G. W. Hammett, Conservative Discontinuous Galerkin Schemes for Nonlinear Fokker-Planck Collision Operators, In preparation

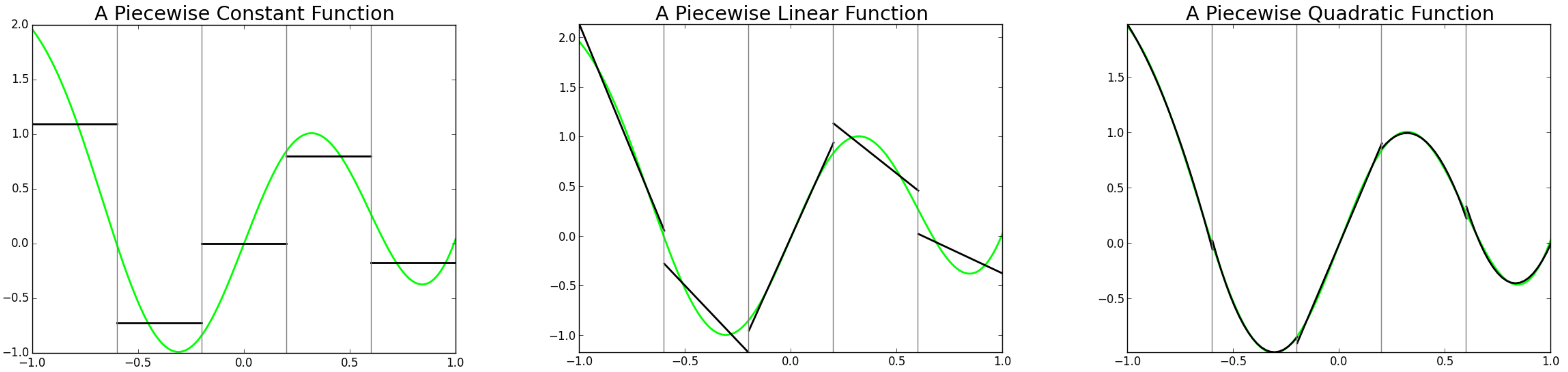

*https://gkyl.readthedocs.io/

# The Discontinuous Galerkin Finite Element Method

We choose to use the discontinuous Galerkin framework to discretize the full phase space of the Vlasov-Maxwell system because it combines aspects of

-Finite elements: high order accuracy and ability to handle complicated geometries

-Finite volume: locality of data and stability enforcing limiters



# The Discrete Vlasov Equation

- What does the discontinuous Galerkin discretization of the Vlasov equation look like?

$$\frac{\partial f_s}{\partial t} + \nabla \cdot (\mathbf{v} f_s) + \nabla_v \cdot (\mathbf{F}_s f_s) = 0$$

  - Consider a phase space mesh $\mathcal{T}$ with cells $K_j \in \mathcal{T}, j = 1, \ldots, N$.
  - Then the problem formulation is, find $f_h \in \mathcal{V}_h^p$, such that for all $K_j \in \mathcal{T}$,

$$\int_{K_j} w \frac{\partial f_h}{\partial t}\, d\mathbf{z} + \oint_{\partial K_j} w^- \mathbf{n} \cdot \hat{\mathbf{F}}\, dS - \int_{K_j} \nabla_z w \cdot \alpha_h f_h\, d\mathbf{z} = 0$$

$$f_h(\mathbf{z}, t) = \sum_n^{N_p} F_n(t) w_n(\mathbf{z}) \qquad \mathcal{V}_h^p = \{v : v|_{K_j} \in \mathbf{P}^p, \forall K_j \in \mathcal{T}\},$$

Upwind fluxes are used for the streaming term and a relaxed global Lax-Friedrichs flux is used for the acceleration

$$\mathbf{n} \cdot \hat{\mathbf{F}} = \frac{1}{2} \mathbf{n} \cdot \left( \boldsymbol{\alpha}_h^+ (f_h^+ + f_h^-) - \boldsymbol{\tau}(f^+ - f^-) \right)$$

$$\text{where } \tau = \max_{\mathcal{T}} \left( \frac{q}{m} \mathbf{E}_h + \frac{q}{m} \mathbf{v} \times \mathbf{B}_h \right)$$

note that the phase space flux is continuous at corresponding surface interfaces

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<u>Conserves</u>
Number density
Energy
L2 norm of the distribution decays monotonically

<u>Cost Mitigation</u>
Lua-JIT & C++
Computer algebra pre-generated kernels
MPI + MPI-3 shared memory
Reduced basis sets

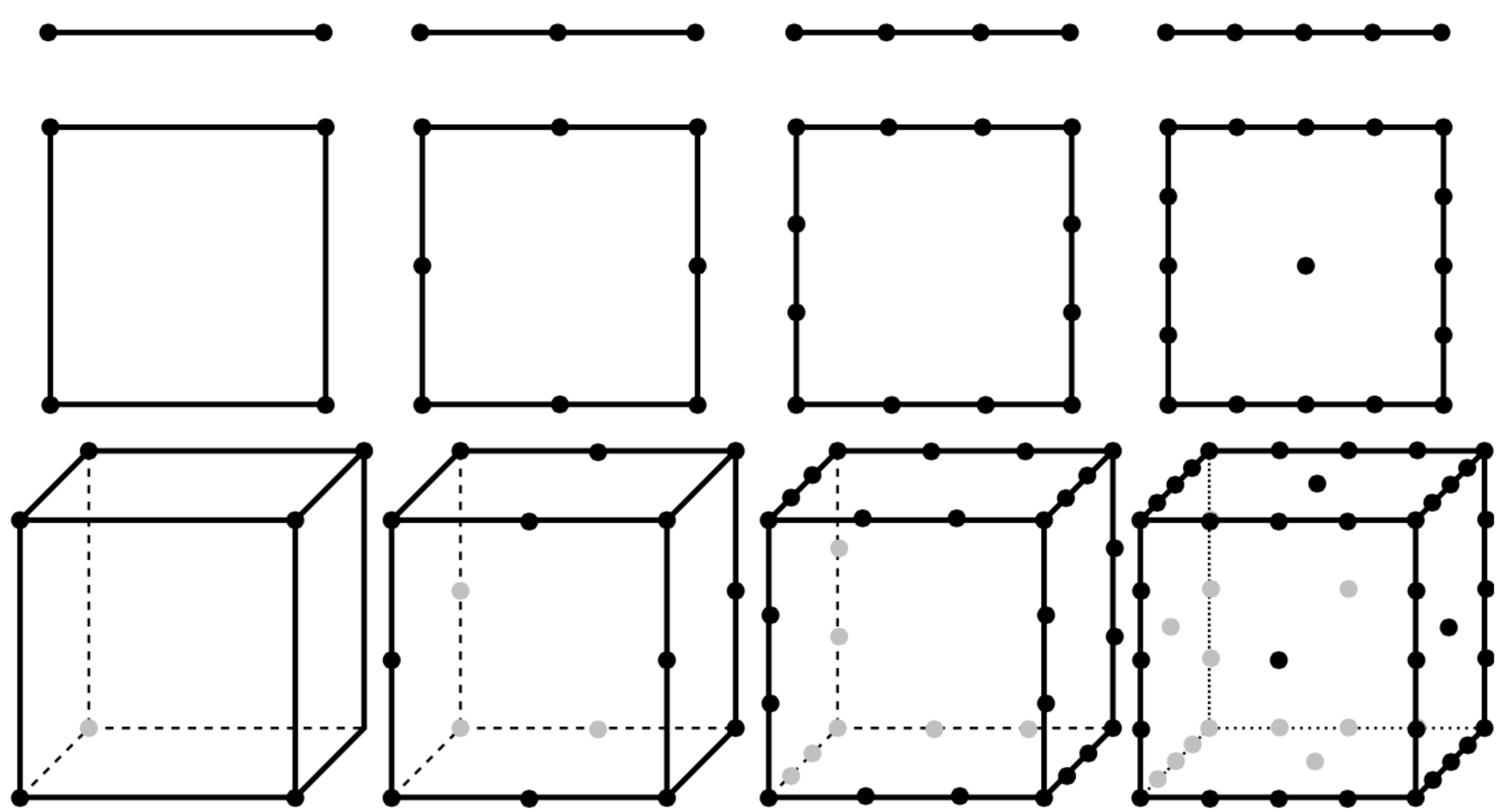
# Serendipity Finite Elements [Arnold & Awanou (2011)]

- We use the Serendipity finite element basis set to mitigate the curse of dimensionality cost but retain the same formal convergence order

- Monomials with super-linear degree greater than p are dropped

- Consider $d = 3$, $p = 2$:
  Retain $x^2yz$
  Drop $x^2y^2z$

$$N_p = \sum_{i=0}^{\min(d,p/2)} 2^{n-i} \binom{d}{i} \binom{p-i}{i}$$



# Collision Operator in Gkeyll

$$\frac{\partial f_s}{\partial t} + \nabla \cdot (\boldsymbol{v} f_s) + \nabla_{\boldsymbol{v}} \cdot (\boldsymbol{a}_s f_s) = \left(\frac{\partial f_s}{\partial t}\right)_c$$

Full Fokker-Planck form
$$\left(\frac{\partial f_s}{\partial t}\right)_c = C[f] = -\frac{\partial}{\partial v_i}\left(\langle \Delta v_i \rangle_s f_s\right) + \frac{1}{2}\frac{\partial^2}{\partial v_i \partial v_j}\left(\left\langle \Delta v_i \Delta v_j \right\rangle_s f_s\right)$$

Lenard-Bernstein (Dougherty)

$$\langle \Delta v_i \rangle_s = -\nu_{ss}(v_i - u_{s,i}) - \sum_{r \neq s} \nu_{sr}(v_i - u_{sr,i})$$

$$\langle \Delta v_i \Delta v_j \rangle_s = 2\nu_{ss} v_{th,s}^2 \delta_{ij} + \sum_{r \neq s} 2\nu_{sr} v_{th,sr}^2 \delta_{ij}$$

# Collision Operator in Gkeyll

$$\frac{\partial f_s}{\partial t} + \nabla\cdot(\boldsymbol{v} f_s) + \nabla_{\boldsymbol{v}}\cdot(\boldsymbol{a}_s f_s) = \left(\frac{\partial f_s}{\partial t}\right)_c$$

Full Fokker-Planck form

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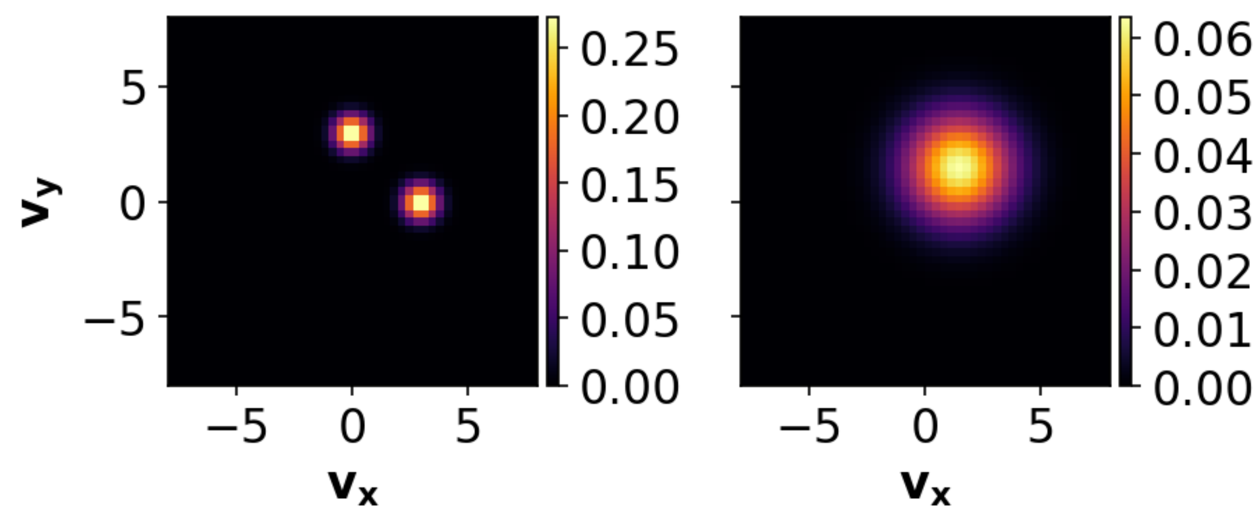
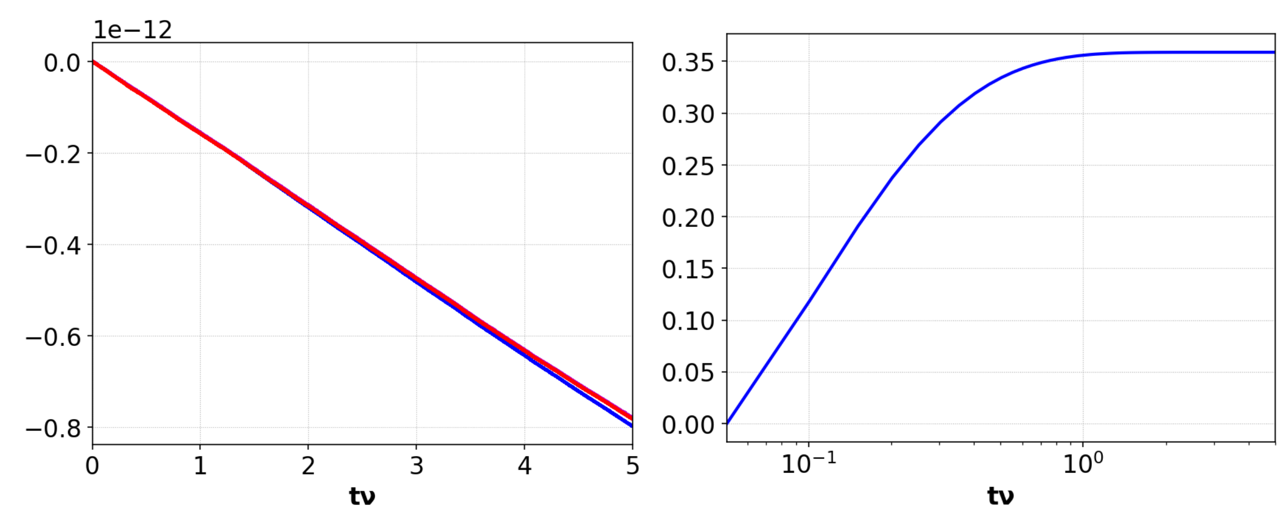



Conserves
- Number density
- Momentum
- Energy

Satisfies
- Discrete H-Theorem
- Maximum entropy is numerical Maxwellian



# Magnetic Pumping [Lichko et al ApJL (2017)]

$$L_x = 200\sqrt{2}\pi\rho_e,\ |v^e_{max}| = 8\sqrt{2}v^e_{th}$$

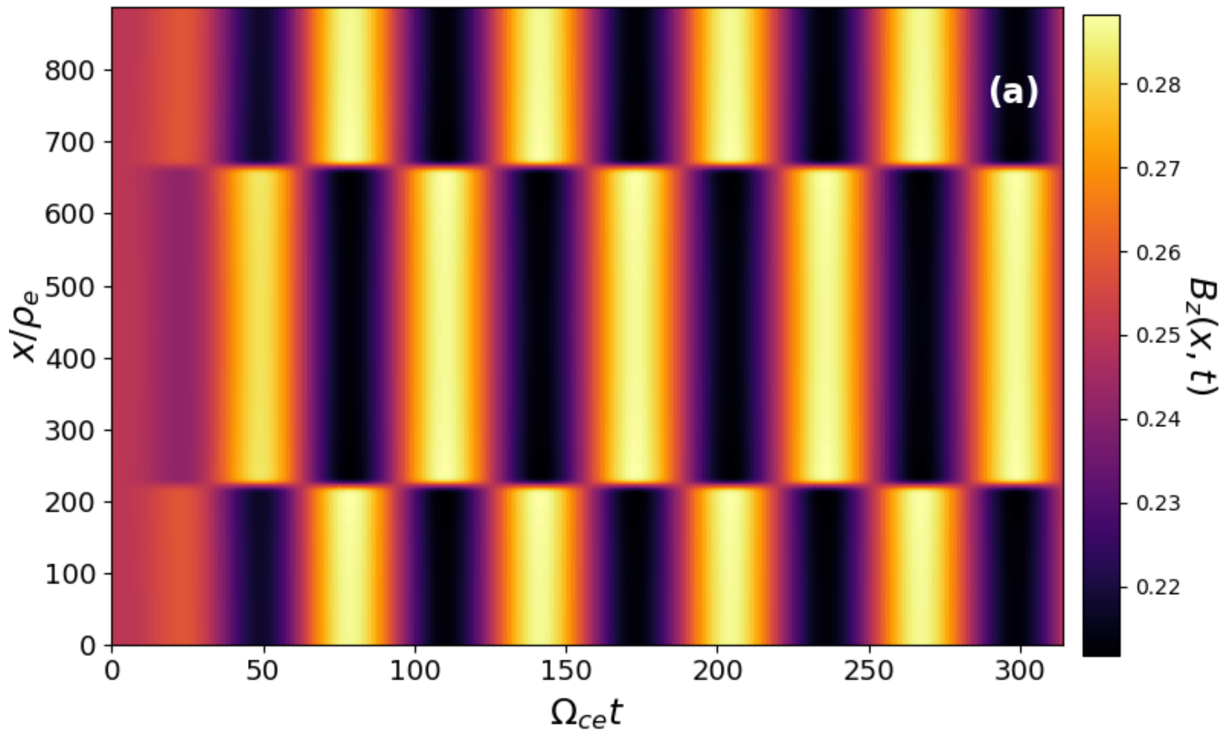
$$T_e/T_i = 1, \beta = 0.01, \mathbf{B}(t=0) = B_0\hat{\mathbf{z}}$$

$$m_i/m_e = 1836, c/v^e_A = 4$$

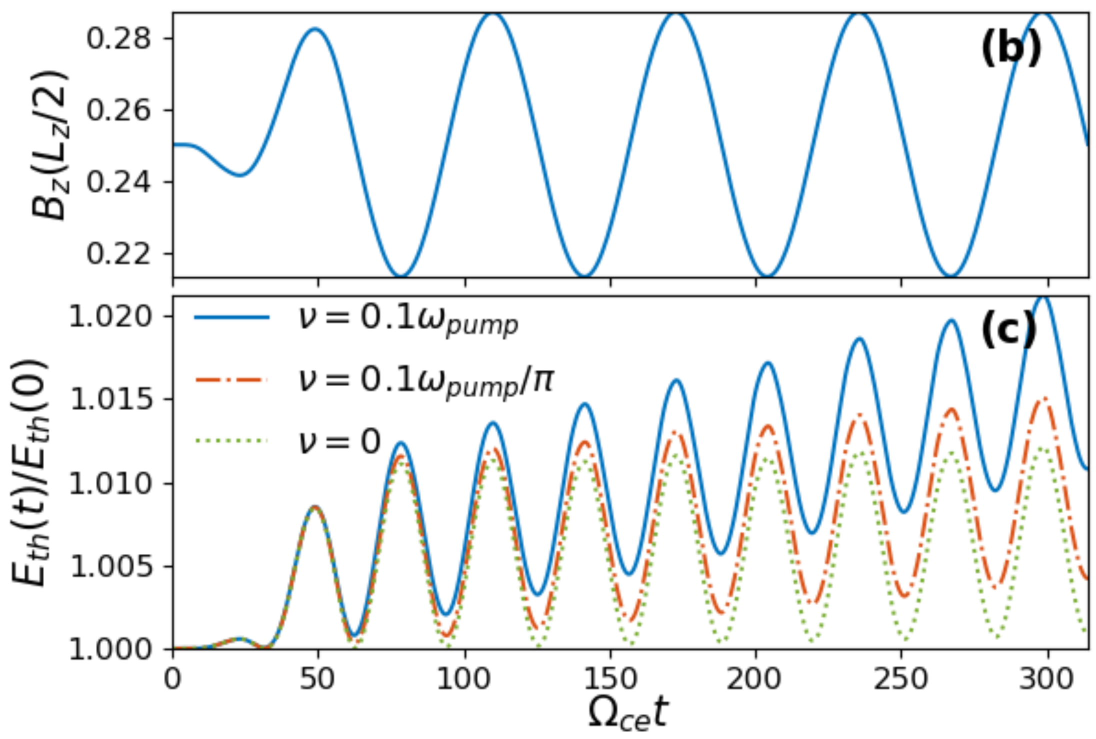
$$(n_x, n^3_{vs}) = (256, 24^3)\ p = 2$$

$$\boldsymbol{J} = \hat{\mathbf{y}} J_0 \sin^2\left(0.5\pi\ \min(1, \omega_{\text{ramp}} t)\right) \sin(\omega_{\text{pump}} t) \left[\exp\left(-\frac{(x-x_1)^2}{2\sigma_J^2}\right) - \exp\left(-\frac{(x-x_2)^2}{2\sigma_J^2}\right)\right]$$

$$\omega_{pump} = 0.1\Omega_{ce},\ \sigma_J = L_x/n_x,\ J_0/enc = v^e_A/2c$$




Magnetic Field



# Field Particle Correlations

# Vlasov-Poisson [Howes, Klein, & Li (2017)]

$$\frac{\partial f_s}{\partial t} + v\frac{\partial f_s}{\partial x} - \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_s}{\partial v} = 0$$

$$f_s(x,v,t) = f_{s0}(v) + \delta f_s(x,v,t)$$

Separation useful in some cases but not necessary

$$\frac{\partial \delta f_s}{\partial t} = -v\frac{\partial \delta f_s}{\partial x} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_{s0}}{\partial v} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial \delta f_s}{\partial v}$$

Multiply by $mv^2/2$ and integrate to obtain the energy equation

$$\frac{\partial W_s}{\partial t} = \int dx \int dv \frac{1}{2} m_s v^2 \left[ -v\frac{\partial \delta f_s}{\partial x} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_{s0}}{\partial v} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial \delta f_s}{\partial v} \right]$$

$$\frac{\partial W_s}{\partial t} = \int dx \int dv \, \frac{1}{2} m_s v^2 \frac{\partial f_s}{\partial t}$$

# Vlasov-Poisson [Howes, Klein, & Li (2017)]

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$$\frac{\partial W_s}{\partial t} = \int dx \int dv \frac{1}{2}m_s v^2 \left[-v\frac{\partial \delta f_s}{\partial x} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_{s0}}{\partial v} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial \delta f_s}{\partial v}\right]$$

$$\frac{\partial W_s}{\partial t} = \int dx \int dv\ \frac{1}{2}m_s v^2 \frac{\partial f_s}{\partial t}$$

In terms of fluid moments

$$\frac{1}{2}\frac{\partial P}{\partial t} = -\nabla \cdot \frac{\mathbf{Q}}{2} + qn\mathbf{u}\cdot\mathbf{E}$$

# Vlasov-Poisson [Howes, Klein, & Li (2017)]

$$\frac{\partial f_s}{\partial t} + v\frac{\partial f_s}{\partial x} - \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_s}{\partial v} = 0$$

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$$\frac{\partial W_s}{\partial t} = \int dx \int dv \frac{1}{2} m_s v^2 \left[\cancel{-v\frac{\partial \delta f_s}{\partial x}} + \cancel{\frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_{s0}}{\partial v}} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial \delta f_s}{\partial v}\right]$$

$$\frac{\partial W_s}{\partial t} = -\int dx \int dv\ q_s \frac{v^2}{2}\frac{\partial \delta f_s(x,v,t)}{\partial v} E(x,t) = \int dx \int dv\ q_s v \delta f_s(x,v,t) E(x,t)$$

$$\frac{\partial W_s}{\partial t} = -\int dx \frac{\partial \phi}{\partial x}\int dv\ q_s v \delta f_s = \int dx\ j_s E$$

# Vlasov-Poisson [Howes, Klein, & Li (2017)]

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The field particle correlation: $C_1(v,t,\tau) = C_\tau \left( -q_s \frac{v^2}{2} \frac{\partial \delta f_s(x_0,v,t)}{\partial v}, E(x_0,t) \right)$

Alternative for cases in which df/dv is difficult to compute

$$C_2(v,t,\tau) = C_\tau \left( q_s v \delta f_s(x_0,v,t), E(x_0,t) \right)$$

In discrete form

$$C_1(v,t_i,\tau) = \frac{1}{N} \sum_{j=i}^{i+N} q_s \frac{v^2}{2} \frac{\partial \delta f_{sj}(v)}{\partial v} E_j$$

Note that f or δf can be used

$$t_j \equiv t(j\Delta t) \qquad \tau = N\Delta t,$$

# Vlasov-Poisson [Howes, Klein, & Li (2017)]

$$\frac{\partial W_s}{\partial t} = -\int dx \int dv\; q_s \frac{v^2}{2} \frac{\partial \delta f_s(x,v,t)}{\partial v} E(x,t) = \int dx \int dv\; q_s v \delta f_s(x,v,t) E(x,t)$$

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Note that f or δf can be used

$$t_j \equiv t(j\Delta t) \qquad \tau = N\Delta t,$$

Other quantities that will appear:

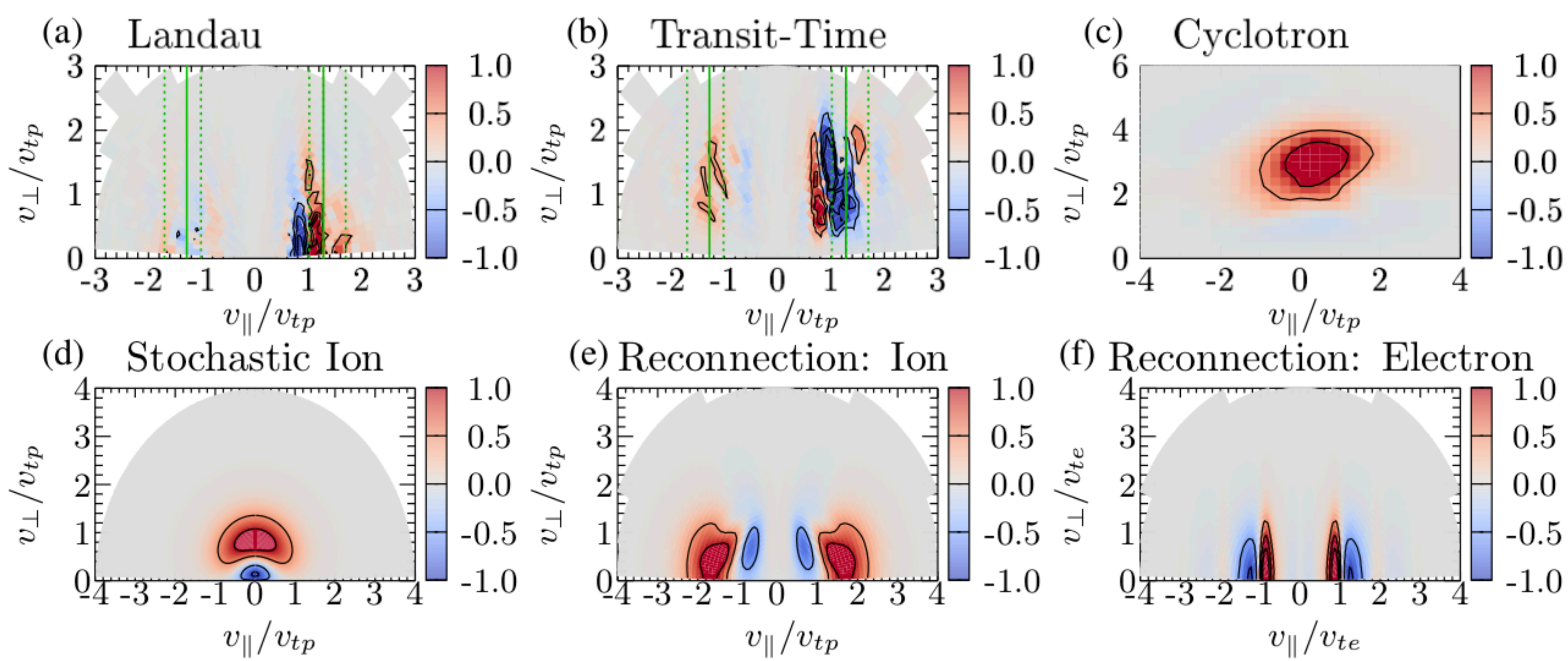
$$\Delta w_s = \int_0^t (\partial w / \partial t') dt'$$

# Field-Particle Correlations Parallel and Perpendicular

In three dimensions, the field-particle correlation can be split into parallel and perpendicular portions. The interpretation of the correlation is simplest in gyrotropic space.

$$C_E(\mathbf{v}) = C(-q\frac{v^2}{2}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}}, \mathbf{E}) = C(-q\frac{v_\parallel^2}{2}\frac{\partial f(\mathbf{v})}{\partial v_\parallel}, E_\parallel) + C(-q\frac{v_\perp^2}{2}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}_\perp}, \mathbf{E}_\perp)$$

$$C_{E_\parallel}(v_\parallel, \mathbf{v}_\perp) = C(-q\frac{v_\parallel^2}{2}\frac{\partial f(\mathbf{v})}{\partial v_\parallel}, E_\parallel) \qquad C_{E_\perp}(v_\parallel, \mathbf{v}_\perp) = C(-q\frac{v_\perp^2}{2}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}_\perp}, \mathbf{E}_\perp)$$



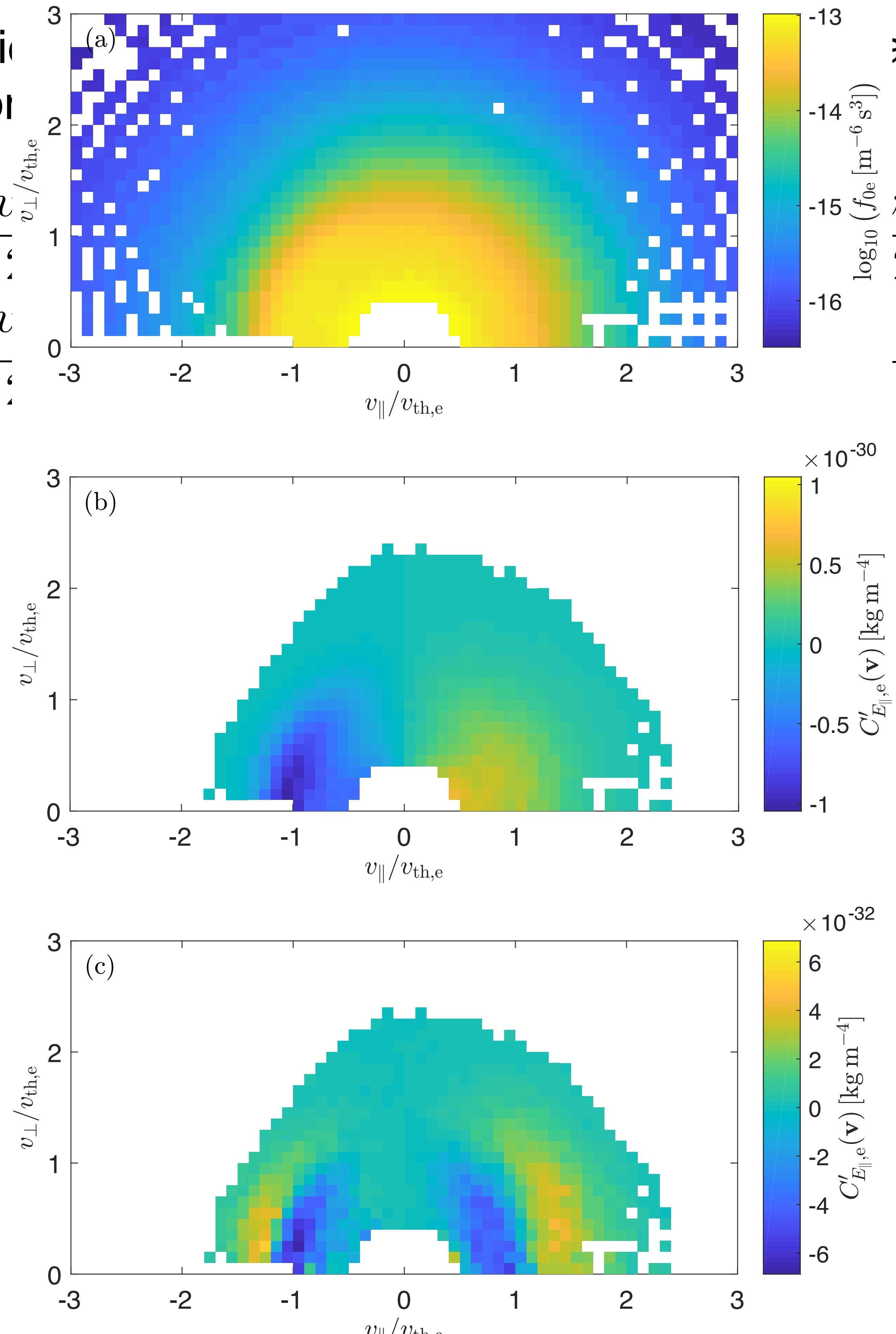

Gyrotropic $(v_\parallel, v_\perp)$ velocity-space signatures: `AstroGK` turbulence simulation for (a) Landau damping using $C_{E_\parallel}$ and (b) transit-time damping using $C_{\delta B_\parallel}$; HVM turbulence simulation for (c) cyclotron damping using $C_{E_\perp}$; theoretical prediction for (d) stochastic ion heating using $C_{E_\perp}$; `AstroGK` strong-guide field reconnection simulation for (e) ion and (f) electron energization using $C_{E_\parallel}$.

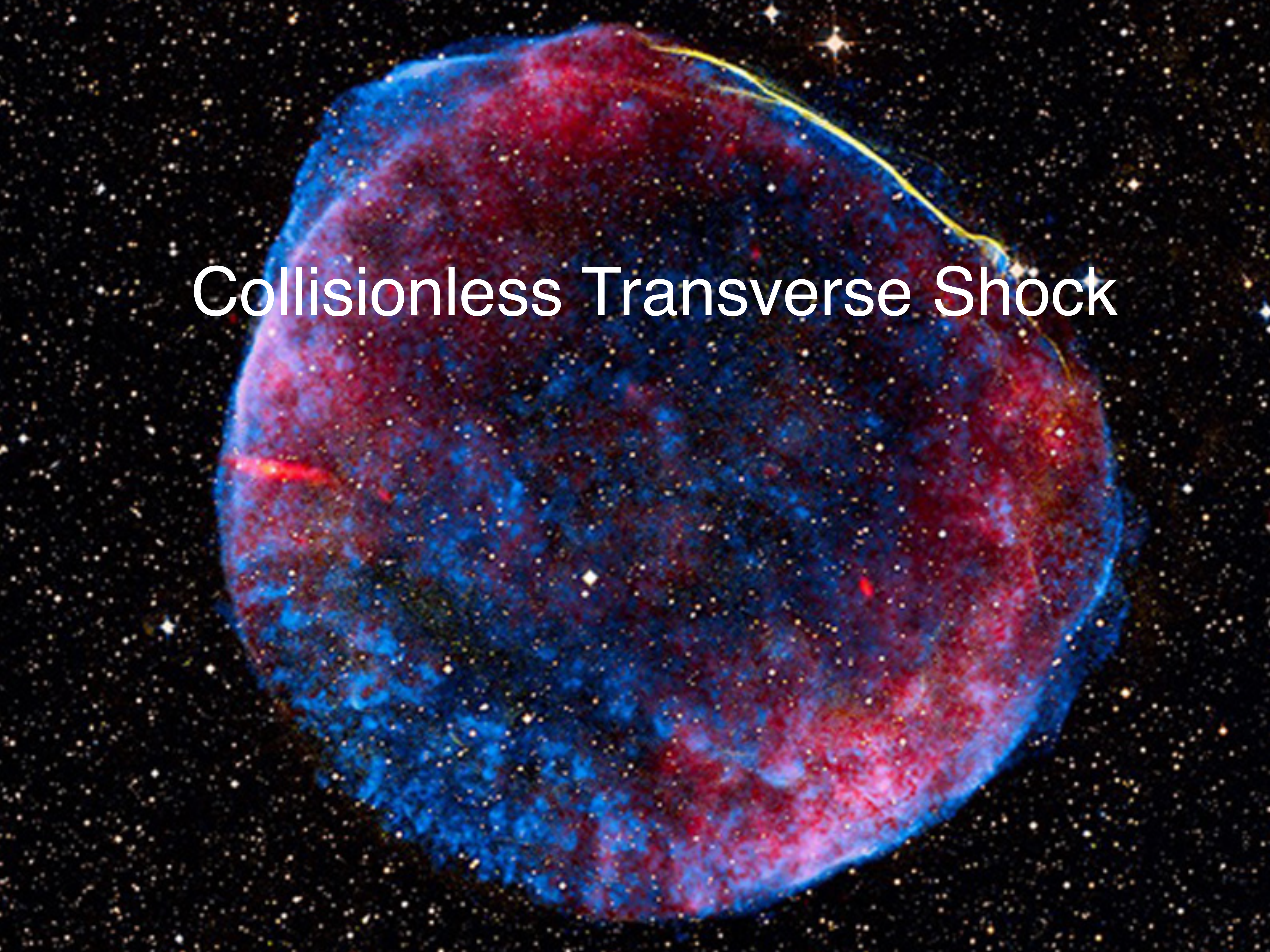
# MMS FPC [Chen et al Nature Comm (2019)]

In three dimensions, the fi … allel and perpendicular portions. The interpretatio … space.

$$C_E(\mathbf{v}) = C(-q \; \ldots \; \frac{v_\perp^2}{2}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}_\perp}, \mathbf{E}_\perp)$$

$$C_{E_\parallel}(v_\parallel, \mathbf{v}_\perp) = C(-q \; \ldots \; -q\frac{v_\perp^2}{2}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}_\perp}, \mathbf{E}_\perp)$$




Collisionless Transverse Shock

# Transverse Shock Simulation Setup

Injection method to initialize the shock

$$L_x = 512 d_e,\ v_{0s} = 2v_A$$

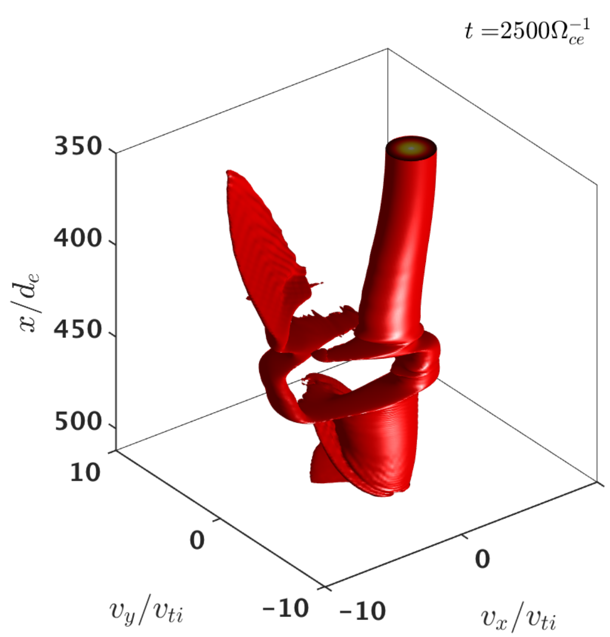
$$T_e/T_i = 4, \beta_e = 1, \mathbf{B} = B_0\hat{\mathbf{z}}$$

$$m_i/m_e = 1836, c/v^e_{th} = 10$$

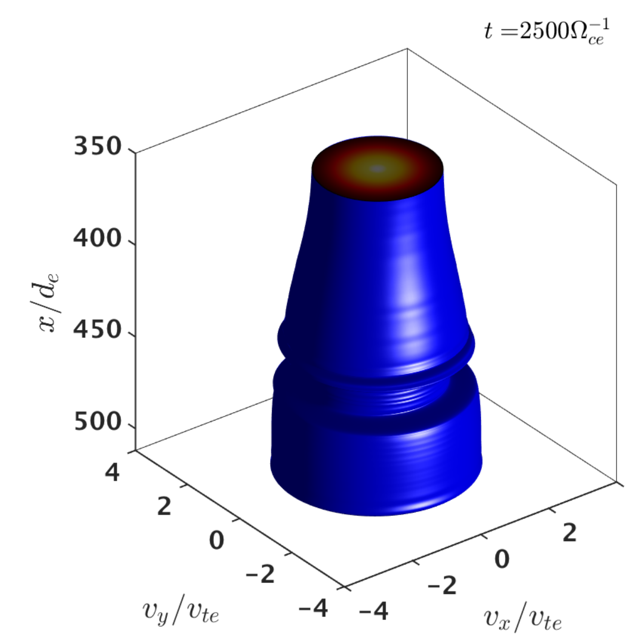
$$(n_x, n^2_{vs}) = (512, 48^2),\ p = 2$$

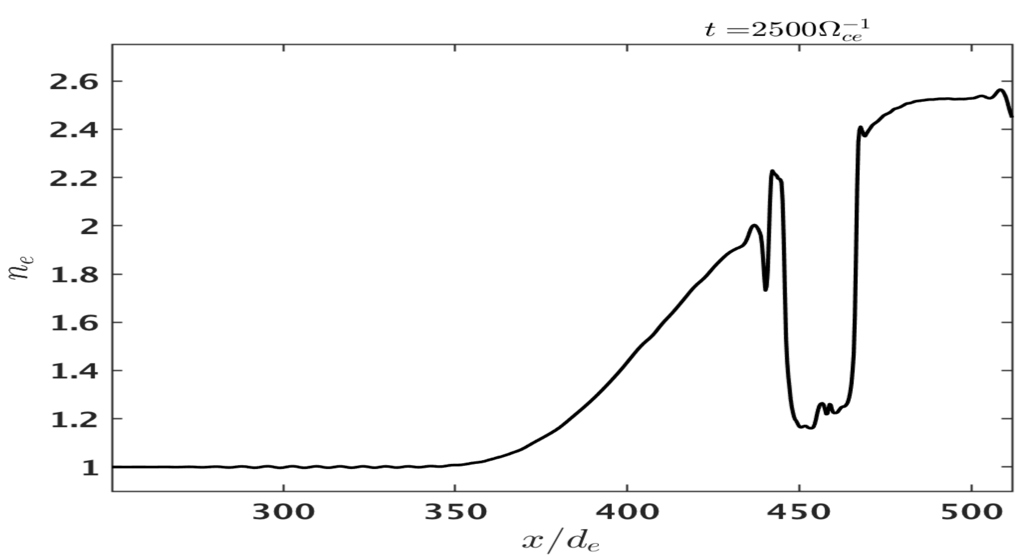
$$|v^i_{max}| = 12v^i_{th},\ |v^e_{max}| = 6v^e_{th}$$

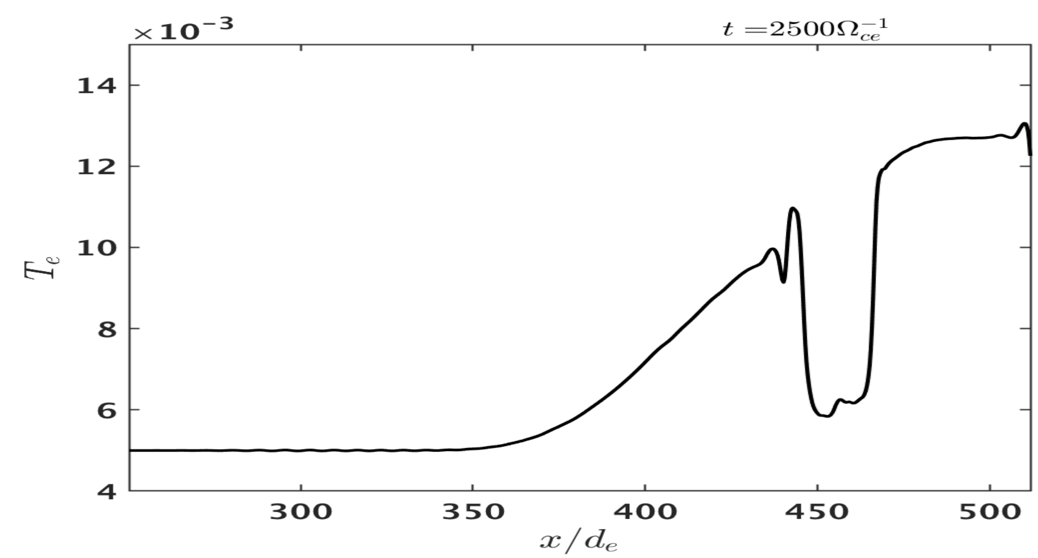
$f_i$



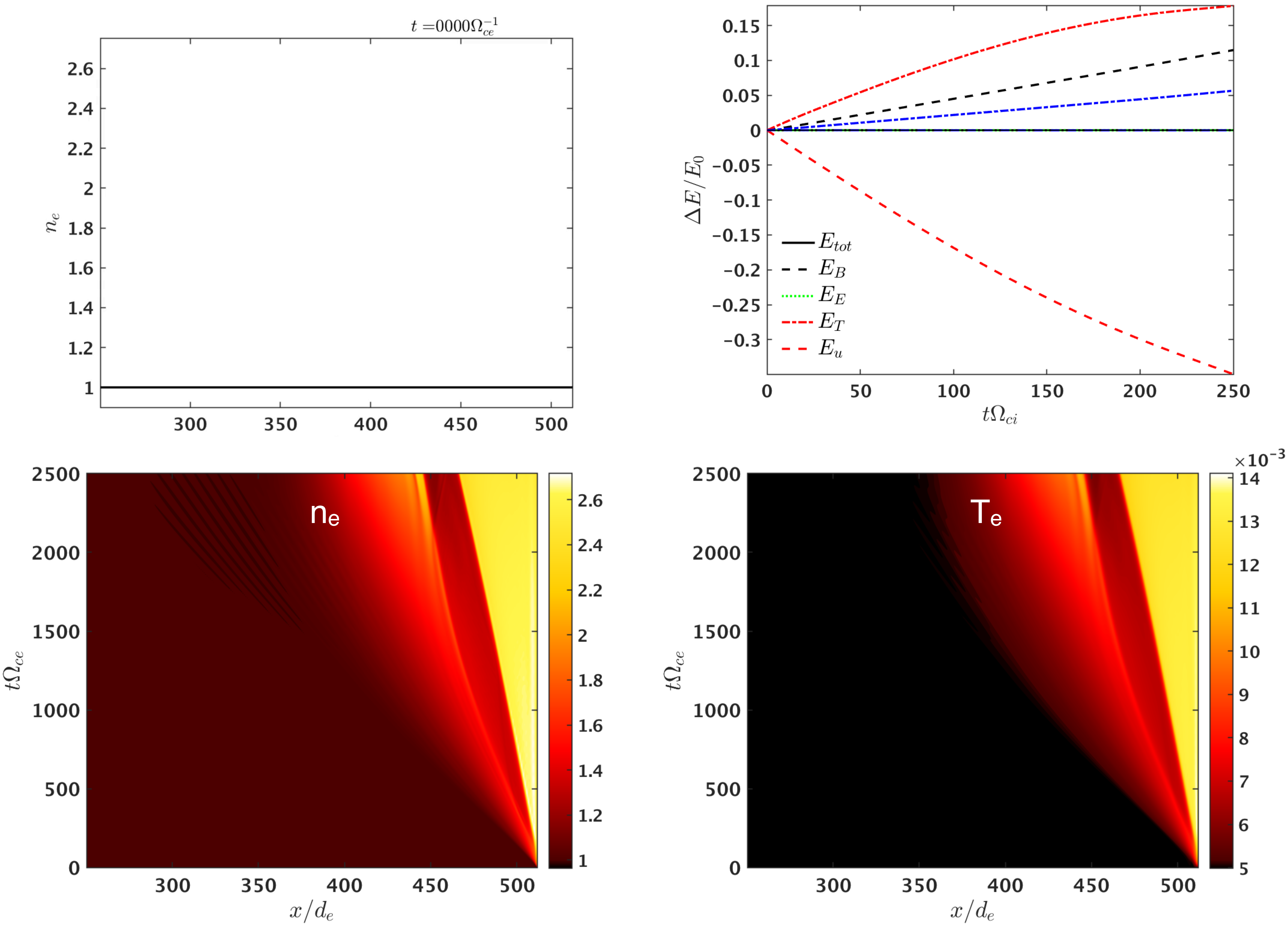

$f_e$



# Transverse Shock Evolution



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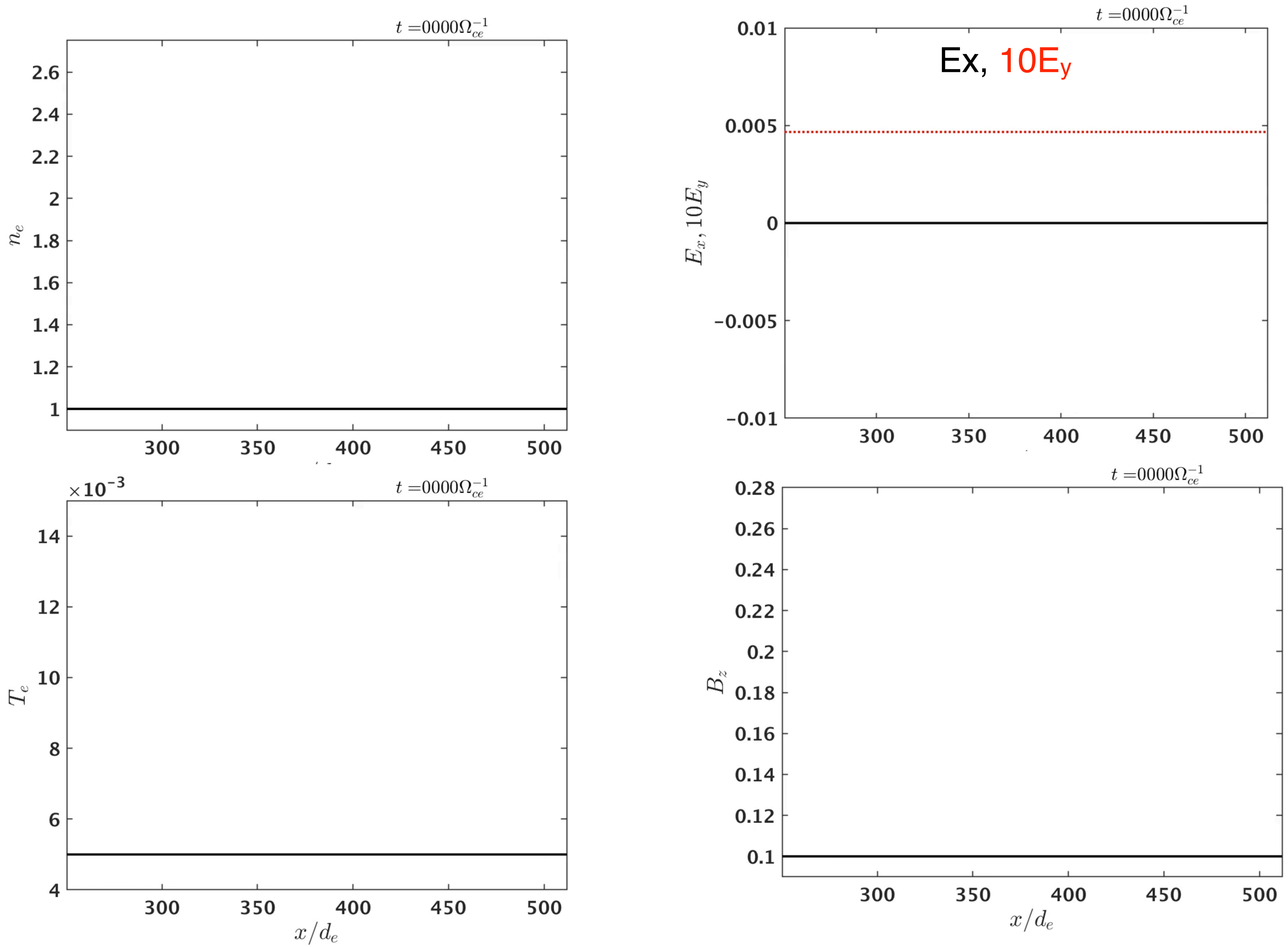
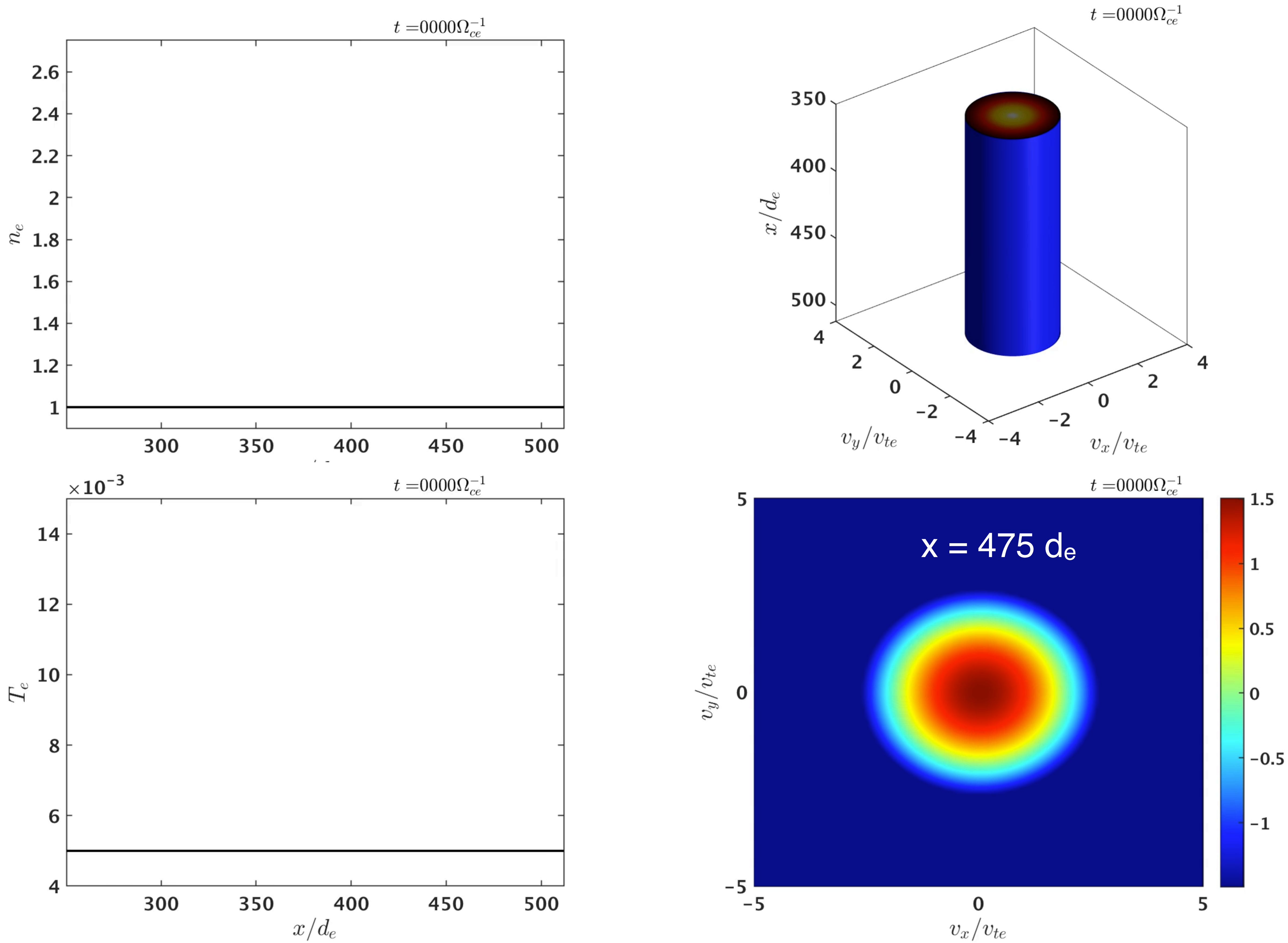
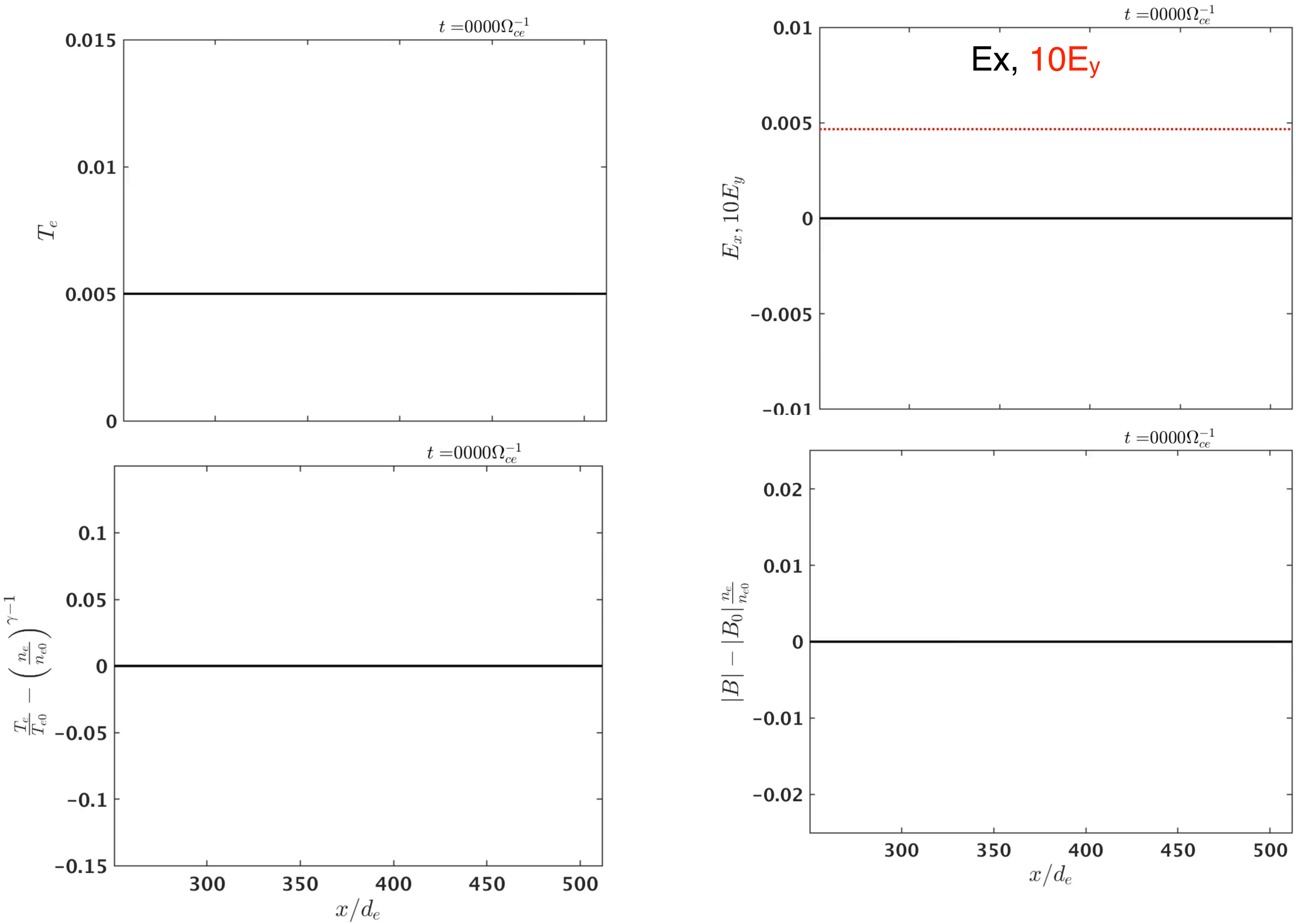


# Transverse Shock Evolution, Electrons

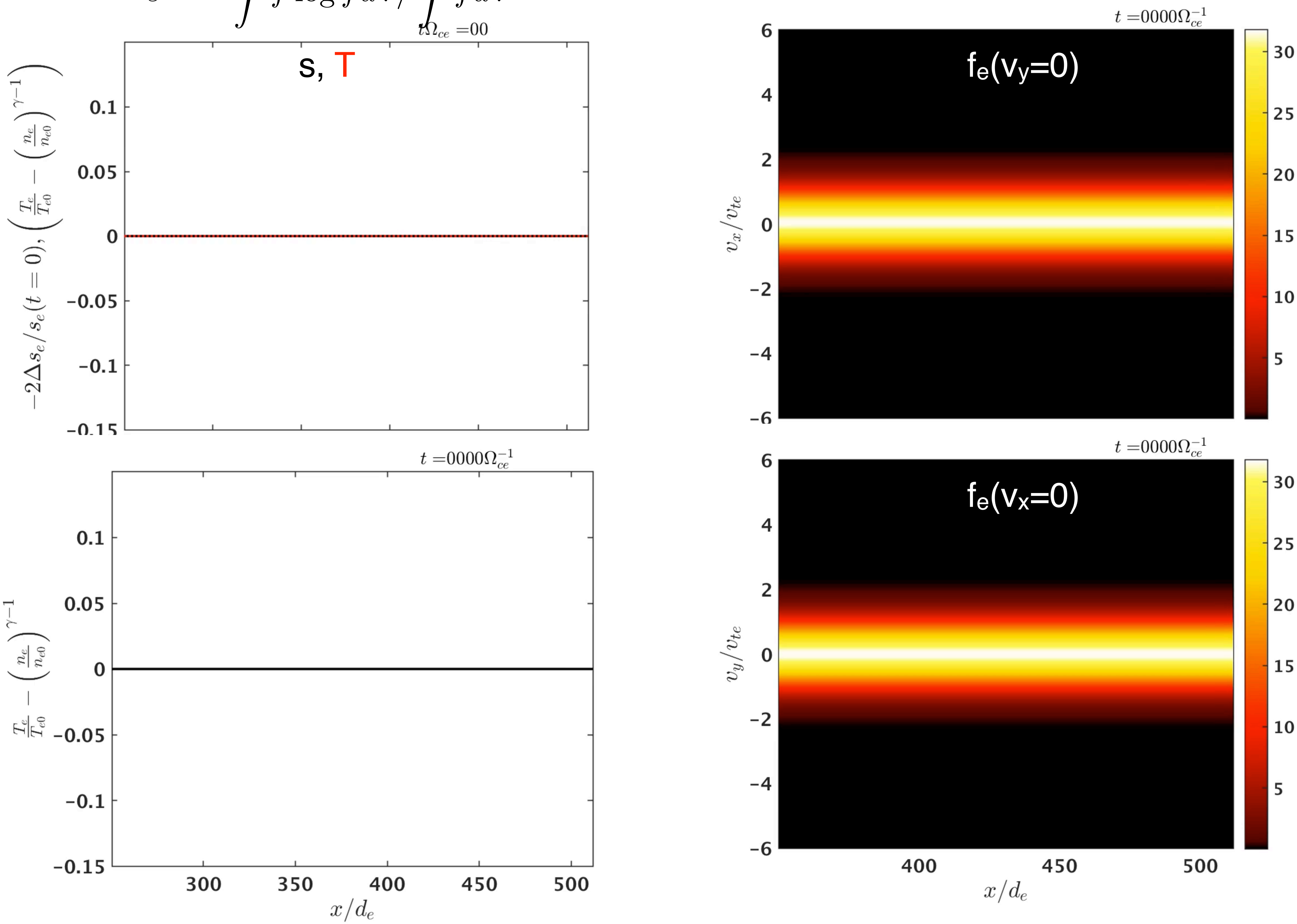


# Transverse Shock Non-Adiabatic Evolution, Electrons

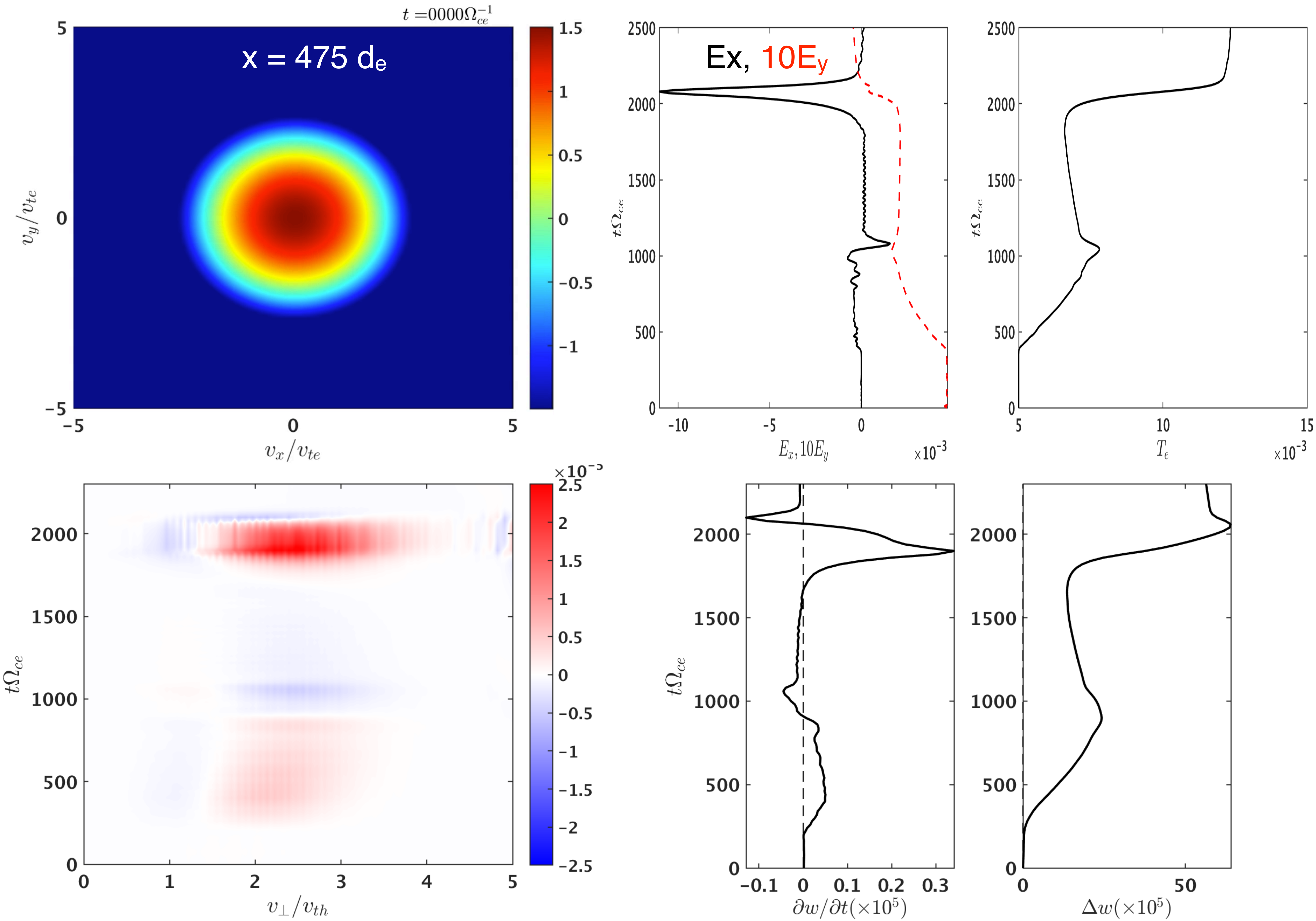


# Transverse Shock Non-Adiabatic Evolution, Electrons

$$s = -\int f \log f \, d\mathbf{v} / \int f \, d\mathbf{v}$$



# Transverse Shock Non-Adiabatic Evolution, Electrons



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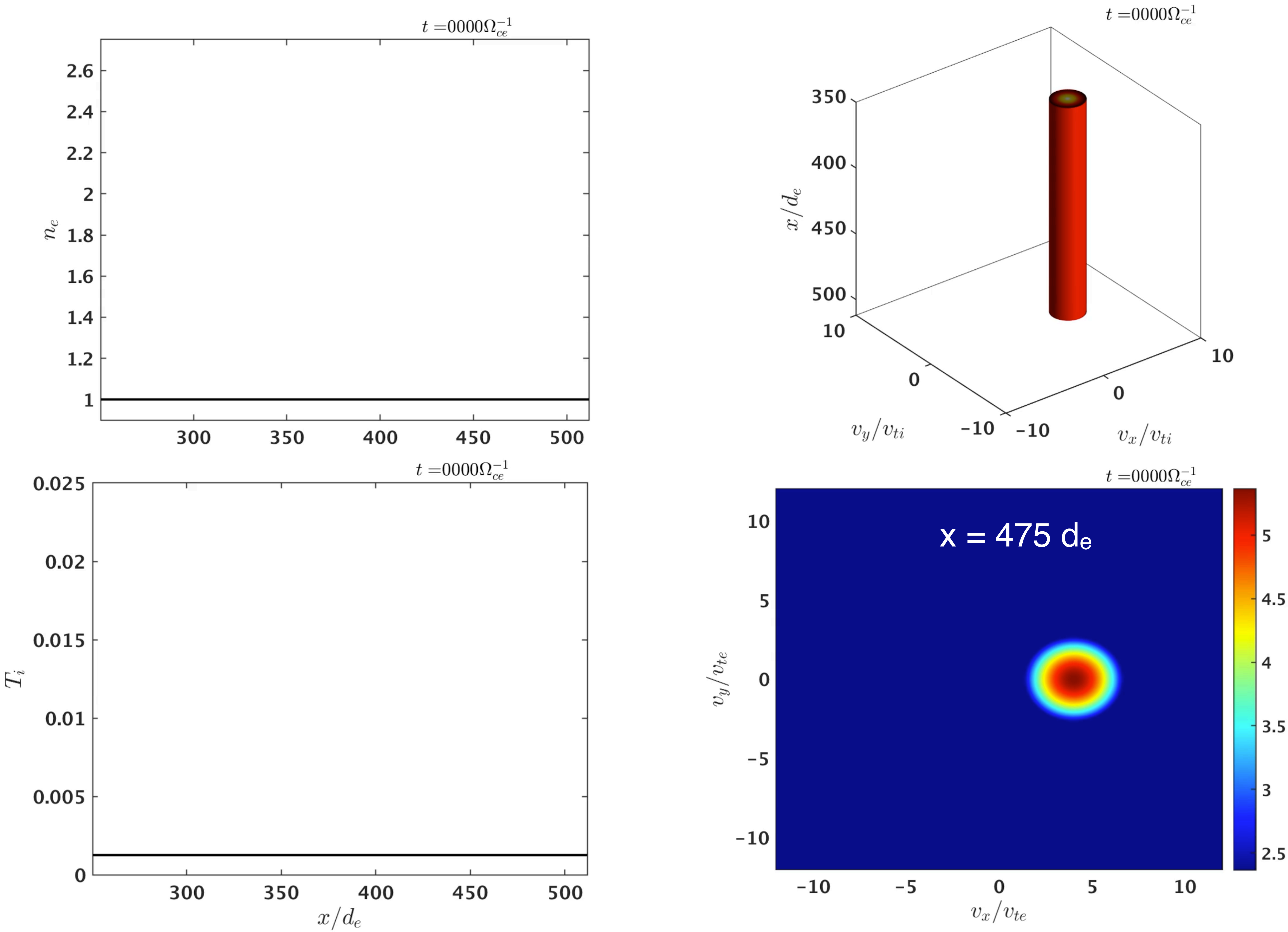
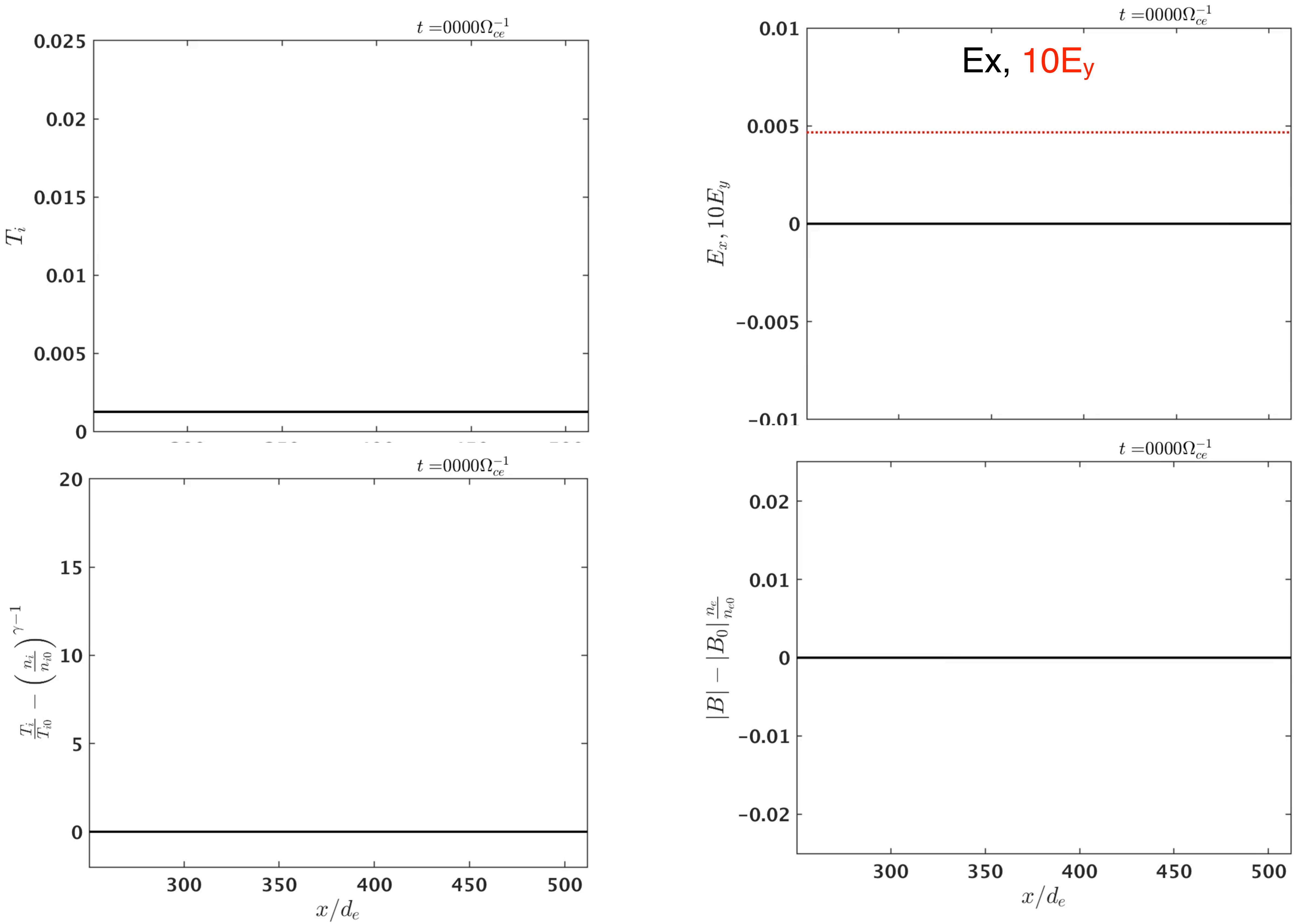
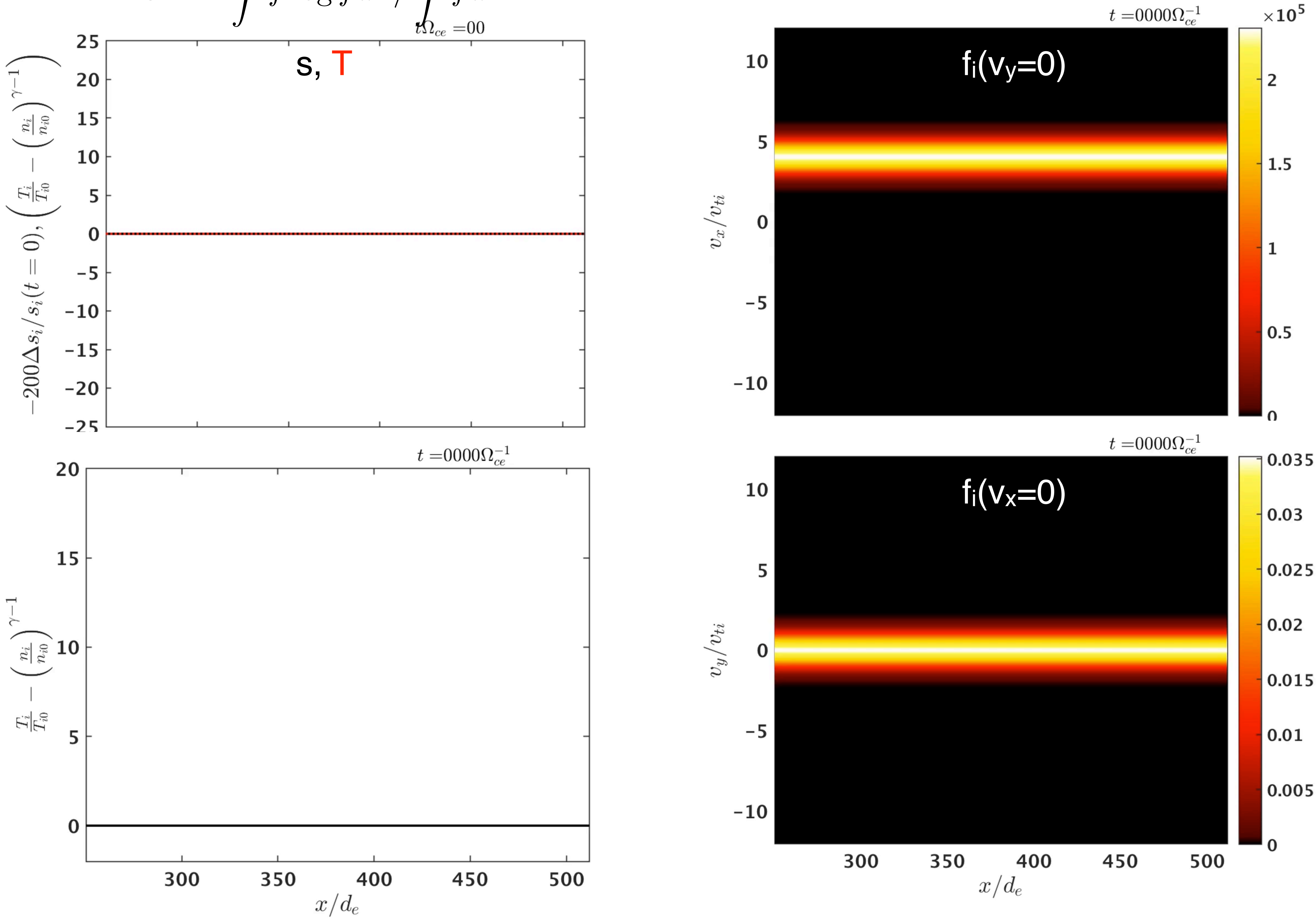


# Transverse Shock Non-Adiabatic Evolution, Ions

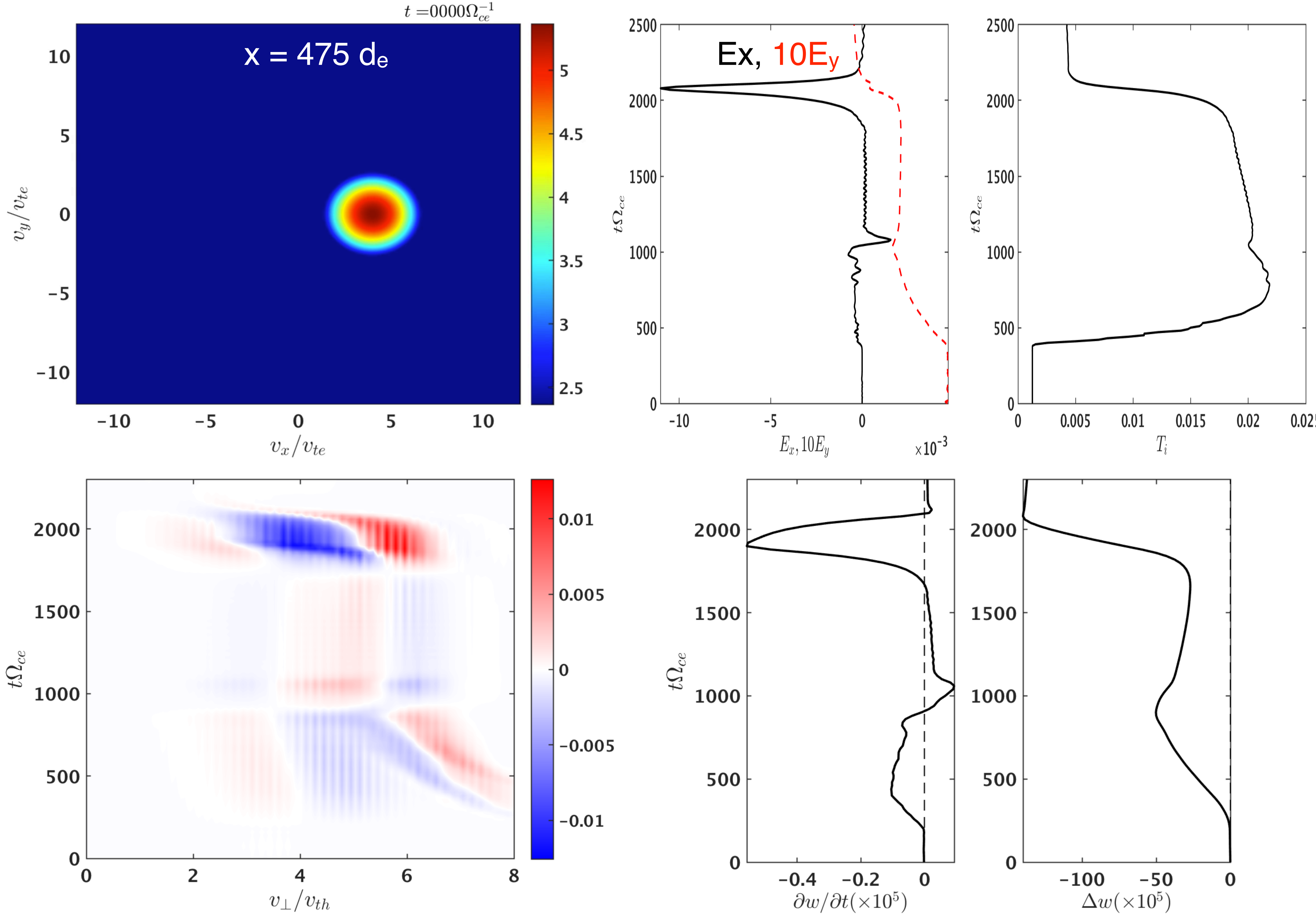


# Transverse Shock Non-Adiabatic Evolution, Ions

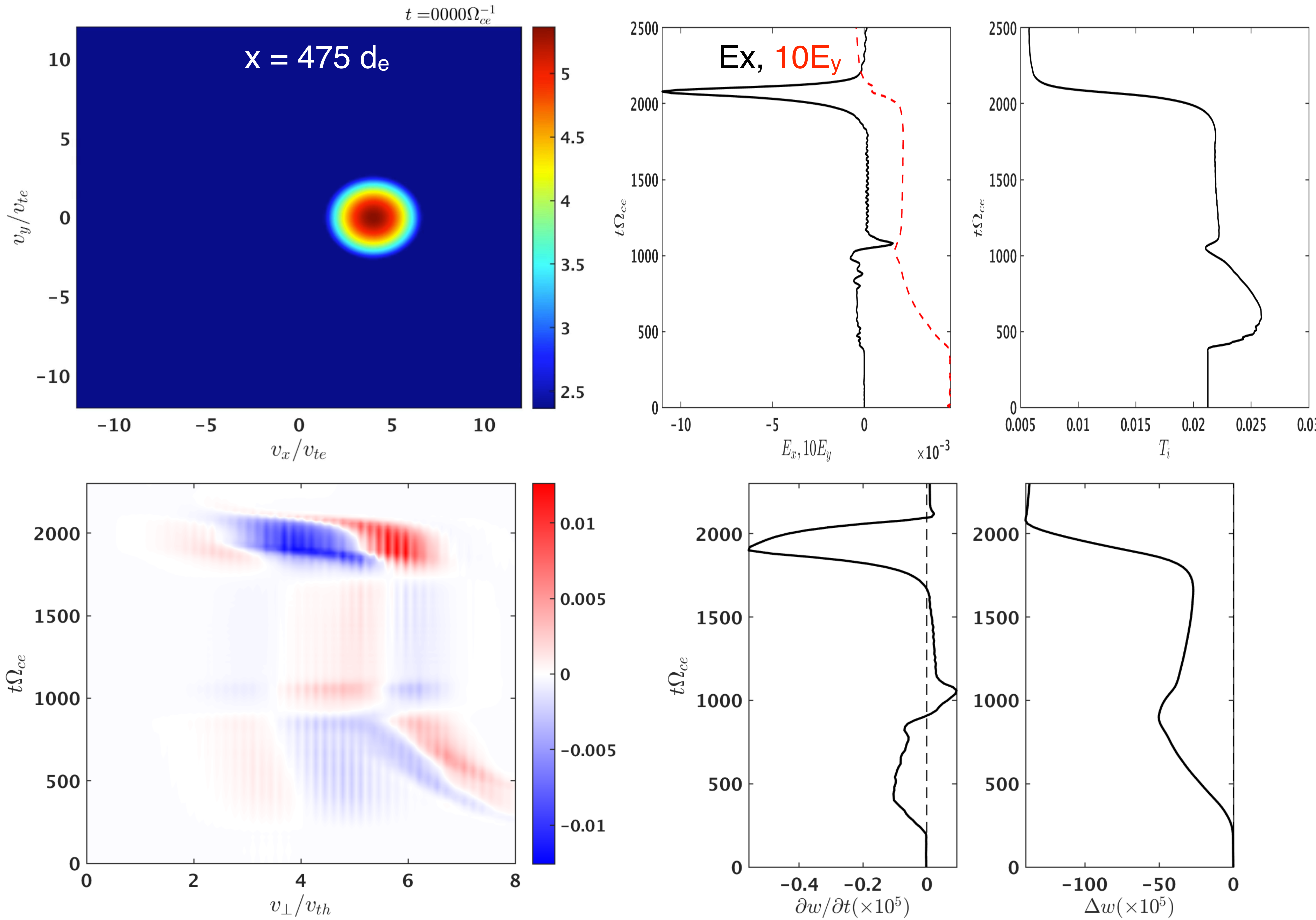
$$s = -\int f \log f \, d\mathbf{v} \Big/ \int f \, d\mathbf{v}$$



# Transverse Shock Non-Adiabatic Evolution, Ions



# Transverse Shock Non-Adiabatic Evolution, Ions



# Conclusions

- Using the continuum Vlasov-Maxwell simulation portion of Gkeyll, we have successfully performed fully kinetic simulations of collisional magnetic pumping and a 1x-2v, collisionless transverse shock.

- The location and source of electron entropy production and heating in the transverse shock has been identified.

- We have employed the field-particle correlation analysis technique to identify the phase space structure of the energy transfer between the fields and particles.

- In most cases, the dominant source of particle energization seen with the field-particle correlation technique is cyclotron-like dissipation, which is most intense at the shock crossing.